NASA TT F-10,622

QUANTUM HYDRODYNAMIC EQUATIONS FOR MULTI-COMPONENT SYSTEMS

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Translation of "K uravneniyam kvantovoy gidrodinamiki dlya mnogokomponentnykh sistem."
Vestnik Moskogo Universiteta, Seriya Fiziki i Astronomii, Vol. 21, No. 5, pp. 63-69, 1966.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON D.C. FEBRUARY 1967

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Equations of hydrodynamics are obtained on the basis of quantum statistics for multi-component systems in an ideal liquid approximation.

The present article applies the generalization of results obtained in (Ref. 1) to the quantum case. We also confine ourselves to obtaining equations of hydrodynamics in the ideal liquid approximation. This problem was studied in (Ref. 2, 3, and 4) for one-component systems. /63*

We shall investigate a system (which interacts by means of central forces) of Fermi- and Bose particles, whose Hamiltonian has the following form in the representation of secondary quantization

$$H = \sum_{1\leq i\leq M}\int\psi_i^+(t,r)\left(-\frac{\hbar^2}{2m_i}\right)\Delta\psi_i(t,r)\,d\vec{r} - \sum_{1\leq i\leq M}\lambda_i\int\psi_i^+(t,r)\psi_i(t,r)\,d\vec{r} + \\ + \sum_{1\leq i\leq M}\int U_i(t,r)\psi_i^+(t,r)\psi_i(t,r)\,d\vec{r} + \\ + \frac{1}{2}\sum_{1\leq i,j\leq M}\int\psi_i^+(t,r)\psi_j^+(t,r')\Phi_{ij}(|r-r'|)\psi_j(t,r')\psi_i(t,r)\,d\vec{r}d\vec{r}',$$

where the indices i, j = 1, 2, ..., M pertain to different types of particles, and the operators $\psi_i(t, r)$ and $\psi_i^+(t, r')$ satisfy the commutation conditions in the Heisenberg representation

$$\psi_i(t,r)\psi_j^+(t,r') \pm \psi_j^+(t,r')\psi_i(t,r) = \Delta(i-j)\,\delta(r-r'), \\ \psi_i\psi_j \pm \psi_j\psi_i = 0, \qquad \psi_i^+\psi_j^+ \pm \psi_j^+\psi_i^+ = 0. \tag{2}$$

The upper sign corresponds to Fermi particles of one type, and the lower sign corresponds to Bose particles and to different (i ≠ j) fermions. Summation with respect to spins is trivial and is omitted everywhere.

In order to obtain the equations of hydrodynamics, we require the expressions for the derivatives with respect to time of the mean density $n_i(t, r)$, the mean flux $\varepsilon(t, r)$ of the particles and of the mean energy per particle $\varepsilon(t, r)$. These quantities may be determined in the customary way:

$$n_i(t,r) = \langle\psi_i^+(t,r)\psi_i(t,r)\rangle, \tag{3}$$

$$j_i^\alpha = \frac{i\hbar}{2}\left\langle\frac{\partial\psi_i^+}{\partial r^\alpha}\psi_i(t,r) - \psi_i^+(t,r)\frac{\partial\psi_i}{\partial r^\alpha}\right\rangle. \tag{4}$$ /64

* Numbers in the margin indicate pagination in the original foreign text.

$$n(t,r)\varepsilon(t,r) = -\sum_{1\leq i\leq M}\frac{\hbar^2}{4m_i}\langle\Delta\psi_i^+\psi_i+\psi_i^+\Delta\psi_i\rangle+$$
$$+\frac{1}{2}\sum_{1\leq i,j\leq M}\int\Phi_{ij}(|r-r'|)\langle\psi_i^+(t,r)\psi_j^+(t,r')\psi_j(t,r')\psi_i(t,r)\rangle\,d\vec{r}', \qquad (5)$$

where the means are taken with a certain unbalanced statistical operator, generally speaking. Summation with respect to spins is trivial and is omitted everywhere.

By employing (1) and (2) we may readily obtain the equations of motion for operator wave functions:

$$i\hbar\frac{\partial\psi_i(t,r)}{\partial t} = -\frac{\hbar^2}{2m_i}\Delta\psi_i(t,r)-(\lambda_i-U_i)\psi_i(t,r)+$$
$$+\sum_{1\leq k\leq M}\int\Phi_{ik}(|r-r'|)\psi_k^+(t,r')\psi_k(t,r')\psi_i(t,r)\,d\vec{r}', \qquad (6)$$

$$i\hbar\frac{\partial\psi_i^+(t,r)}{\partial t} = \frac{\hbar^2}{2m_i}\Delta\psi_i^+(t,r)+(\lambda_i-U_i)\psi_i^+(t,r)-$$
$$-\sum_{1\leq k\leq M}\int\Phi_{ik}(|r-r'|)\psi_i^+(t,r)\psi_k^+(t,r')\psi_k(t,r')\,d\vec{r}'. \qquad (7)$$

Let us now calculate the derivatives with respect to time from (3), (4), and (5). Employing (6) and (7) and performing the transformations in the same way as in (Ref. 4), we obtain

$$m_i\frac{\partial n_i(t,r)}{\partial t}+\sum_{1\leq\beta\leq 3}\frac{\partial j_i^\beta}{\partial r^\beta}=0, \qquad (8)$$

$$\frac{\partial j_i^\alpha(t,r)}{\partial t} = \frac{\hbar^2}{4m_i}\frac{\partial(\Delta n_i)}{\partial r^\alpha}-\frac{\hbar^2}{2m_i}\sum_\beta\frac{\partial}{\partial r^\beta}\left\langle\frac{\partial\psi_i^+}{\partial r^\alpha}\frac{\partial\psi_i}{\partial r^\beta}+\frac{\partial\psi_i^+}{\partial r^\alpha}\frac{\partial\psi_i}{\partial r^\beta}\right\rangle-$$
$$-\frac{\partial U_i}{\partial r^\alpha}n_i(t,r)-\frac{1}{2}\sum_k\int\frac{\partial\Phi_{ik}(|R|)}{\partial R^\alpha}\{D_{ik}(t,r,-R)+D_{ki}(t,r-R,R)\}\,d\vec{R}. \qquad (9)$$

$$\frac{\partial(n\varepsilon)}{\partial t} = \sum_i\frac{i\hbar^3}{8m_i^2}\Delta\langle\Delta\psi_i^+\cdot\psi_i-\psi_i^+\cdot\Delta\psi_i\rangle-\sum_i\frac{1}{m_i}\sum_\beta j_i^\beta\frac{\partial U_i}{\partial r^\beta}+$$
$$+\sum_i\frac{i\hbar^3}{4m_i^2}\sum_\beta\frac{\partial}{\partial r^\beta}\left\langle\frac{\partial\psi_i^+}{\partial r^\beta}\cdot\Delta\psi_i-\Delta\psi_i^+\cdot\frac{\partial\psi_i}{\partial r^\beta}\right\rangle+$$
$$+\sum_{i,k}\frac{1}{m_i}\sum_\beta\frac{\partial}{\partial r^\beta}\int\Phi_{ik}(|R|)G_{ik}^\beta(t,r,R)\,d\vec{R}+ \qquad (10)$$
$$+\sum_{i,k}\sum_\beta\int\frac{\partial\Phi_{ik}}{\partial R^\beta}\left\{\frac{1}{m_i}G_{ik}^\beta(t,r,-R)+\frac{1}{m_k}G_{ki}^\beta(t,r-R,R)\right\}d\vec{R},$$

where

$$D_{ik}(t, r, r'-r) = \langle \psi_i^+(t, r)\psi_k^+(t, r')\psi_k(t, r')\psi_i(t, r)\rangle = D_{ki}(t, r', r-r')$$

and /65

$$G_{ik}^{\alpha}(t, r, r'-r) = \frac{i\hbar}{4}\Big\langle \psi_i^+(t, r)\psi_k^+(t, r')\psi_k(t, r')\frac{\partial\psi_i(t, r)}{\partial r^{\alpha}} - \\ -\frac{\partial\psi_i^+(t, r)}{\partial r^{\alpha}}\psi_k^+(t, r')\psi_k(t, r')\psi_i(t, r)\Big\rangle.$$

Before obtaining the equations of hydrodynamics, let us examine the state of statistical equilibrium in the case of $U_i = 0$, which is characterized by the customary parameters -- density of the number of particles of all components $n_i(t, r)$, temperature $\theta(t, r)$, velocity of the system as a whole $\vec{V}(t, r)$. For the state of statistical equilibrium, means of the type $< \ldots > \ldots n_i, \ldots, \ldots, \theta$, V may be expressed by transformation of the operator functions $\psi_i \to e^{\frac{im_i Vr}{h}}\psi_i$ using the mean values $< \ldots > \ldots n_i, \ldots, \theta$ in the case of $\vec{V} = 0$. We obtain

$$j_i^{\alpha} = m_i n_i V^{\alpha}, \quad n\varepsilon = n\varepsilon_0 + \frac{\rho V^2}{2}, \tag{11}$$

where

$$n = \sum_i n_i \text{ и } \rho = \sum_i m_i n_i, \text{ а } \varepsilon_0(\ldots n_i \ldots, \theta)$$

is the mean energy per one particle for the state of statistical equilibrium of a liquid at rest. It is apparent that in this case all the diffusion fluxes equal

$$I_i^{\alpha} = j_i^{\alpha} - n_i m_i V^{\alpha} = 0. \tag{12}$$

Let us examine a quantity of the type

$$A_{ik} = \langle (D_1\psi_i^+(t, r))(D_2\psi_k^+(t, r'))(D_3\psi_k(t, r'))(D_4\psi_i(t, r))\rangle,$$

where D_v represent linear combinations of constants and differentiation operators with respect to spatial variables.

Due to the spatial uniformity of the state of statistical equilibrium, we have

$$A_{ik} = A_{ik}(r'-r \mid \ldots n_i \ldots, \theta, V).$$

In hydrodynamics, the quantities

$$A_{ik}(t, r, R) = <(D_1\psi_i^+(t, r))(D_2\psi_k^+(t, r'))(D_3\psi_k(t, r'))(D_4\psi_i(t, r)) \tag{13}$$

and the outer field $U_i(t, r)$ change very little in the case of spatial and time translations. We shall assume that quantities of the type (13) differ to a sufficiently small extent from the corresponding equilibrium values $A_{ik}(R|\ \ldots n_i(t,\ r),\ \ldots,\ \theta(t,\ r),\ \ldots,\ U_i(t,\ r),\ \ldots,\ V(t,\ r)$. The smaller these values, the smaller the gradients of the quantities $n_i(t,\ r)$, $\theta(t,\ r)$, $U_i(t,\ r)$ and $\vec{V}(t,\ r)$, where $\vec{V}$ and θ may be expressed by

$$V^\alpha = \frac{j^\alpha}{\rho} = \frac{j_i^\alpha}{\rho_i},\qquad j^\alpha = \sum_i j_i^\alpha.\quad \varepsilon_0 = \varepsilon_0(\ldots,n_i,\ldots,\Theta). \tag{14}$$

For the formal expression of the assumptions which have been advanced, it is advantageous to introduce the small parameter μ. Then the means of the type under consideration and the outer field assume the following form /66

$$U_i(t,r) = \widetilde{U}_i(\tau,\xi),\qquad A_{ik}(t,r,r'-r) = \widetilde{A}_{ik}(\tau,\xi,R;\mu),\qquad \tau=\mu t,\quad \vec{\xi}=\mu\vec{r}. \tag{15}$$

In view of the assumption which has been advanced regarding the small deviation from statistical equilibrium, we have

$$\widetilde{A}_{ik}(\tau,\xi,R;\mu) = \widetilde{A}_{ik}^{(0)}(\tau,\xi,R) + \mu\widetilde{A}_{ik}^{(1)}(\tau,\xi,R) + \mu^2\ldots, \tag{16}$$

where

$$\widetilde{A}_{ik}^{(0)}(\tau,\xi,R) = \widetilde{A}_{ik}(R|\ldots\widetilde{n}_i(\tau,\xi)\ldots,\widetilde{\theta}(\tau,\xi),\widetilde{V}(\tau,\xi))$$

and

$$\widetilde{V}^\alpha(\tau,\xi)\ \text{and}\ \widetilde{\theta}(\tau,\xi)$$

are determined in accordance with

$$\widetilde{V}^\alpha = \frac{\widetilde{j}^\alpha}{\rho},\quad \widetilde{n}\widetilde{\varepsilon} = \widetilde{n}\widetilde{\varepsilon}_0(\ldots\widetilde{n}_i\ldots,\widetilde{\theta}) + \frac{\widetilde{\rho}\widetilde{V}^2}{2}. \tag{17}$$

The expression $\widetilde{A}_{ik}^{(1)}$ represents a linear form with respect to the gradients $\widetilde{n}_i(\tau,\ \xi)$ $\widetilde{\theta}(\tau,\ \xi)$, $\widetilde{\vec{V}}(\tau,\ \xi)$, and $U_i\ (\tau,\ \xi)$.

Changing from the variables t, $\vec{r}$ to the variables τ, $\vec{\xi}$ in (8), (9) and (10), we obtain

$$m_i\frac{\partial\widetilde{n}_i(\tau,\xi)}{\partial\tau} + \sum_\beta\frac{\partial\widetilde{j}_i^\beta(\tau,\xi)}{\partial\xi^\beta} = 0, \tag{18}$$

$$\frac{\partial\widetilde{j}^\alpha}{\partial\tau} = \mu^2\sum\frac{\hbar^2}{4m_i}\frac{\partial(\Delta_\xi\widetilde{n}_i)}{\partial\xi^\alpha} - \sum_i\frac{\hbar^2}{2m_i}\sum_\beta\frac{\partial}{\partial\xi^\beta}\left\langle\frac{\partial\psi_i^+}{\partial r^\beta}\frac{\partial\psi_i}{\partial r^\alpha} + \frac{\partial\psi_i^+}{\partial r^\alpha}\frac{\partial\psi_i}{\partial r^\beta}\right\rangle - \tag{19}$$

$$-\sum_i \frac{\partial \widetilde{U}_i}{\partial \xi^\alpha}\widetilde{n}_i - \frac{1}{2\mu}\sum_{i,k}\int \frac{\partial \Phi_{ik}}{\partial R^\alpha}\{D_{ik}(\tau,\xi,-R) + D_{ki}(\tau,\xi-\mu R, R)\}\,d\vec{R}, \tag{19}$$

$$\frac{\partial(\widetilde{n}\widetilde{\varepsilon})}{\partial\tau} = \mu\sum_i \frac{i\hbar^3}{8m_i^2}\Delta_\xi\langle\Delta\psi_i^+\cdot\psi_i - \psi_i^+\Delta\psi_i\rangle -$$

$$-\sum_i \frac{i\hbar^3}{4m_i^2}\sum_\beta \frac{\partial}{\partial\xi\beta}\left\langle \Delta\psi_i^+\frac{\partial\psi_i}{\partial r\beta} - \frac{\partial\psi_i^+}{\partial r\beta}\Delta\psi_i\right\rangle +$$

$$+\sum_{i,k}\frac{1}{m_i}\sum_\beta \frac{\partial}{\partial\xi^\beta}\int \Phi_{ik}\widetilde{G}^\beta_{ik}(\tau,\xi,R)\,d\vec{R} - \sum_i \frac{1}{m_i}\sum_\beta \widetilde{j}^\beta_i\frac{\partial\widetilde{U}_i}{\partial\xi^\beta} + \tag{20}$$

$$+\frac{1}{\mu}\sum_{i,k}\sum_\beta\int \frac{\partial\Phi_{ik}}{\partial R^\beta}\left\{\frac{1}{m_i}\widetilde{G}^\beta_{ik}(\tau,\xi,-R) + \frac{1}{m_k}\widetilde{G}^\beta_{ki}(\tau,\xi-\mu R,R)\right\}d\vec{R}.$$

Let us investigate the following quantitity which is contained in (20)

$$\sum_{i,k}\sum_\alpha\int \frac{\partial\Phi_{ik}}{\partial R^\alpha}\left\{\frac{1}{m_i}\widetilde{G}^\alpha_{ik}(\tau,\xi,-R) + \frac{1}{m_k}\widetilde{G}^\alpha_{ki}(\tau,\xi-\mu R,R)\right\}d\vec{R}.$$

Taking the fact into account that the rapidly decreasing quantity $\frac{\partial\Phi_{ik}}{\partial R^\alpha}$ is /67 contained in the integrand, and performing expansion of $\widetilde{G}^\alpha_{ki}(\tau,\xi-\mu R,R)$ with respect to μR, in accordance with (16), we obtain

$$\sum_{i,k}\sum_\alpha\int \frac{\partial\Phi_{ik}}{\partial R^\alpha}\left\{\frac{1}{m_i}\widetilde{G}^\alpha_{ik}(\tau,\xi,-R) + \frac{1}{m_k}\widetilde{G}^\alpha_{ki}(\tau,\xi-\mu R,R)\right\}d\vec{R} =$$

$$= -\mu\sum_{i,k}\frac{1}{m_k}\sum_{\alpha,\beta}\int \frac{\partial\Phi_{ik}}{\partial R^\alpha}R^\beta\frac{\partial\widetilde{G}^\alpha_{ki}}{\partial\xi^\beta}\,d\vec{R} +$$

$$+\frac{\mu^2}{2}\sum_{i,k}\frac{1}{m_k}\sum_{\alpha,\beta,\gamma}\int \frac{\partial\Phi_{ik}}{\partial R^\alpha}R^\beta R^\gamma\frac{\partial^2\widetilde{G}^\alpha_{ki}}{\partial\xi^\beta\partial\xi^\gamma}\,d\vec{R} + 0(\mu^3).$$

We have changed the summation indices and have employed the symmetry of the expression $\widetilde{G}^\alpha_{ik}(\tau,\xi,-R) + \widetilde{G}^\alpha_{ik}(\tau,\xi,R)$.

We also obtain exactly

$$\sum_{i,k}\int \frac{\partial\Phi_{ik}}{\partial R^\alpha}\{\widetilde{D}_{ik}(\tau,\xi,-R) + \widetilde{D}_{ki}(\tau,\xi-\mu R,R)\}\,d\vec{R} =$$

$$= -\mu\sum_{i,k}\sum_\beta\int \frac{\partial\Phi_{ik}}{\partial R^\alpha}R^\beta\frac{\partial D_{ki}}{\partial\xi^\beta}\,d\vec{R} + 0(\mu^3).$$

Then, disregarding terms of the order μ and higher in (19) and (20), we obtain

$$\frac{\partial \tilde{j}^{\alpha}}{\partial \tau} = \sum_{\beta} \frac{\partial T_{\alpha\beta}}{\partial \xi^{\beta}} - \sum_{i} \frac{\partial \tilde{U}_i}{\partial \xi^{\alpha}} \tilde{n}_i(\tau, \xi), \tag{21}$$

$$\frac{\partial (\tilde{n}\tilde{\varepsilon})}{\partial \tau} = -\sum_{i} \frac{1}{m_i} \sum_{\beta} \tilde{j}_i^{\beta} \frac{\partial \tilde{U}_i}{\partial \xi^{\beta}} + \sum_{\alpha} \frac{\partial S_{\alpha}}{\partial \xi^{\alpha}}, \tag{22}$$

where

$$T_{\alpha\beta} = -\sum_{i} \frac{\hbar^2}{2m_i} \left\langle \frac{\partial \psi_i^{+}}{\partial r^{\alpha}} \frac{\partial \psi_i}{\partial r^{\beta}} + \frac{\partial \psi_i^{+}}{\partial r^{\beta}} \frac{\partial \psi_i}{\partial r^{\alpha}} \right\rangle + \\ + \frac{1}{2} \sum_{i,k} \int \frac{\partial \Phi_{ik}}{\partial R^{\alpha}} R^{\beta} \tilde{D}_{ki}(\tau, \xi, R)\, d\vec{R} \tag{23}$$

and

$$S_{\alpha} = -\sum_{i} \frac{i\hbar^3}{4m_i^2} \left\langle \Delta\psi_i^{+} \frac{\partial \psi_i}{\partial r^{\alpha}} - \frac{\partial \psi_i^{+}}{\partial r^{\alpha}} \Delta\psi_i \right\rangle + \\ + \sum_{i,k} \frac{1}{m_i} \int \Phi_{ik} \tilde{G}_{ik}^{\alpha}(\tau, \xi, R)\, d\vec{R} - \sum_{i,k} \frac{1}{m_k} - \\ - \sum_{i,k} \frac{1}{m_k} \sum_{\beta} \int \frac{\partial \Phi_{ik}}{\partial R^{\beta}} R^{\alpha} \tilde{G}_{ki}^{\beta}(\tau, \xi, R)\, d\vec{R}. \tag{24}$$

All the means are taken with respect to the state of statistical equilibrium which is determined by the parameters $\tilde{n}_i(\tau,\xi)$, $\tilde{\theta}(\tau,\xi)$ and $\tilde{\vec{V}}(\tau,\xi)$. Let us substitute the operator wave functions $\psi_i \rightarrow e^{\frac{im_i Vr}{h}} \psi_i$. In a state of statistical equilibrium of a liquid at rest, all of the means must be invariant with respect to the transformation $\vec{r} \rightarrow -\vec{r}$ and $\vec{k} \rightarrow -\vec{k}$ (k - wave vector). /68

In our approximation, we obtain the following for the quantities $T_{\alpha\beta}$ and S_{α}

$$T_{\alpha\beta} = -\tilde{\rho}\tilde{V}^{\alpha}\tilde{V}^{\beta} - \delta_{\alpha\beta} P(\ldots \tilde{n}_i \ldots, \tilde{\theta}), \tag{25}$$

$$S_{\alpha} = -\tilde{V}^{\alpha}(\tilde{n}\tilde{\varepsilon}_0(\ldots \tilde{n}_i \ldots, \tilde{\theta}) + \frac{\tilde{\rho}\tilde{V}^2}{2} - P(\ldots \tilde{n}_i \ldots, \tilde{\theta}), \tag{26}$$

where

$$P(\ldots \tilde{n}_i \ldots, \tilde{\theta}) = \sum_{i} \frac{\hbar^2}{m_i} \left\langle \frac{\partial \psi_i^{+}}{\partial r^{\alpha}} \frac{\partial \psi_i}{\partial r^{\alpha}} \right\rangle - \\ - \frac{1}{2} \sum_{i,k} \int \frac{\partial \Phi_{ik}}{\partial R^{\alpha}} R^{\alpha} \tilde{D}_{ki}(R \mid \ldots \tilde{n}_i \ldots, \tilde{\theta})\, d\vec{R}. \tag{27}$$

The following relationships may also be employed when deriving (26)

$$\widetilde{G}^{\alpha}_{ik}(R|\ldots\widetilde{n}_i\ldots,\widetilde{\theta},\widetilde{V}) = \frac{m_i\widetilde{V}^{\alpha}}{2}\widetilde{D}_{ik}(R|\ldots\widetilde{n}_i\ldots,\widetilde{\theta})$$

and

$$\widetilde{n}\widetilde{\varepsilon}_0(\ldots\widetilde{n}_i\ldots,\widetilde{\theta}) = \sum_i \frac{\hbar^2}{2m_i}\sum_\beta \left\langle \frac{\partial\psi_i^+}{\partial r^\beta}\frac{\partial\psi_i}{\partial r^\beta}\right\rangle_{\ldots\widetilde{n}_i\ldots,\widetilde{\theta}} +$$
$$+\frac{1}{2}\sum_{i,k}\int \Phi_{ik}\widetilde{D}_{ik}(R|\ldots\widetilde{n}_i\ldots,\widetilde{\theta})\,d\vec{R}.$$

Equations (21) and (22) then assume the form

$$\frac{\partial\widetilde{j}^\alpha}{\partial\tau} = -\sum_\beta \frac{\partial(\widetilde{\rho}\widetilde{V}^\alpha\widetilde{V}^\beta)}{\partial\xi^\beta} - \frac{\partial P(\ldots\widetilde{n}_i\ldots,\widetilde{\theta})}{\partial\xi^\alpha} - \sum_i \frac{\partial\widetilde{U}_i}{\partial\xi^\alpha}\widetilde{n}_i, \quad (28)$$

$$\frac{\partial(\widetilde{n}\widetilde{\varepsilon})}{\partial\tau} = -\sum_\alpha \frac{\partial}{\partial\xi^\alpha}\left\{\widetilde{V}^\alpha\left[(\widetilde{n}\widetilde{\varepsilon}_0(\ldots\widetilde{n}_i\ldots,\widetilde{\theta}) + \frac{\widetilde{\rho}\widetilde{V}^2}{2} + P(\ldots\widetilde{n}_i\ldots,\theta)\right]\right\} - \quad (29)$$
$$-\sum_i\frac{1}{m_i}\sum_\beta \widetilde{j}_i^\beta\frac{\partial\widetilde{U}_i}{\partial\xi^\beta}.$$

In order to have a comprehensive hydrodynamic description of a multi-component system, we must add the expression for diffusion fluxes to the equations obtained. However, it may be readily seen that there is no diffusion in the approximation which is assumed here -- as must be the case. In actuality, we have

$$\widetilde{I}_i^\alpha(\tau,\xi) = {}^{(0)}\widetilde{j}_i - \widetilde{\rho}_i\widetilde{V}^\alpha = 0,$$

since ${}^{(0)}\widetilde{j}_i^\alpha(\ldots\widetilde{n}_i\ldots,\widetilde{\theta},\widetilde{V})$ may be determined according to the state of local equilibrium and in accordance with (14) ${}^{(0)}\widetilde{j}_i^\alpha = \widetilde{\rho}_i\widetilde{V}^\alpha$.

Let us introduce entropy for one molecule $\widetilde{s} = -\dfrac{\partial F(\ldots\widetilde{n}_i\ldots,\widetilde{\theta})}{\partial\widetilde{\theta}}$, where /69

$F(\ldots\widetilde{n}_i,\ldots,\widetilde{\theta})$ is free energy. In exactly the same way as in (Ref. 1), equation (29) may be transformed to the following form

$$\frac{d\widetilde{s}}{d\tau} = \frac{\partial\widetilde{s}}{\partial\tau} + \sum_\alpha \widetilde{V}^\alpha\frac{\partial\widetilde{s}}{\partial\xi^\alpha} = 0. \quad (30)$$

Returning to the variables t, r in equations (18), (28), (30), we obtain the well known system of hydrodynamic equations for an ideal liquid.

Consideration of terms of the order μ in (23) and (24) leads to equations of hydrodynamics with allowance for viscosity, thermal conductivity, and diffusion. This case will be investigated in a subsequent article.

The author would like to express his appreciation to academician N. N. Bogolyubov for his constant attention to this study, and also to

I. A. Kvasnikov and V. D. Kukin for valuable discussions.

REFERENCES

1. Krasnikov, V.A. "Vestnik Moskogo Universiteta", Seriya Fiziki, Astronomii, No. 3, 1966.
2. Bogolyubov, N.N. Prats' Institutu Matematiki, No. 10, 1948.
3. Gurov, K. P. Zhurnal Eksperimental'noy i Teoreticheskoy Fiziki, 18, 110, 1948.
4. Bogolyubov, N.N. K voprosu o gidrodinamike sverkhtekuchey zhidkosti (Hydrodynamics of a Superfluid Liquid). Preprint Ob'yedinennyy Institut Yadernykh Issledovaniy. Dubna, 1963.

Received
January 6, 1965

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